Exam Quantum Field Theory
January 26, 2023
Start: 15:00h End: 17:00h

Write your name and student ID on each sheet.

INSTRUCTIONS. This is a closed-book and closed-notes exam. You are allowed to bring one A4 page written by you, with useful formulas. The exam duration is 2 hours. There is a total of 9 points that you can collect.

NOTE: If you are not asked to **Show your work**, then an answer is sufficient. However, you might always earn more points by answering more extensively (but you can also lose points by adding wrong explanations). If you are asked to **Show your work**, then you should explain your reasoning and the mathematical steps of your derivation in full. Use the official exam paper for *all* your work and ask for more if you need.

USEFUL FORMULAS

For the energy projectors for spin 1/2 Dirac fermions use the normalization without the factor $1/(2m)$.

$$\sum_{r=1,2} u_r(\vec{p})\bar{u}_r(\vec{p}) = \not{p} + m$$

$$\sum_{r=1,2} v_r(\vec{p})\bar{v}_r(\vec{p}) = \not{p} - m$$

$$\{\gamma_5, \gamma^\mu\} = 0, \quad (\gamma^0)^2 = \mathbb{1}, \quad \gamma_5^2 = \mathbb{1}, \quad \gamma_5^\dagger = \gamma_5, \quad \gamma^0\gamma^{\mu\dagger}\gamma^0 = \gamma^\mu$$

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \quad \mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} - g^{\mu\rho}g^{\nu\sigma})$$

$$\gamma^\mu\gamma_\mu = 4\mathbb{1} \quad \gamma^\mu\not{p}\gamma_\mu = -2\not{p} \quad \not{k}\not{p}\not{k} = 2(pk)\not{k} - k^2\not{p}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu) = \mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu) = \mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho) = 0$$

1. (3 points total) Consider the Lagrangian density

$$\mathcal{L} \;=\; \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + \partial_\mu\Phi^\dagger\partial^\mu\Phi - M^2\Phi^\dagger\Phi - \frac{\lambda}{2}\varphi^2\Phi^\dagger\Phi \tag{1}$$

that describes the interaction between a real scalar field (φ) and a complex scalar field (Φ, $\Phi^\dagger$)

a) [**2 points**] Given the general formula for the 4-point correlation function for φ

$$G^{(4)}_\varphi(x_1,x_2,x_3,x_4) = \frac{1}{Z[\lambda=0]}\int\mathcal{D}\varphi\int\mathcal{D}\Phi^\dagger\mathcal{D}\Phi\, e^{i\int d^4x\,\mathcal{L}}\,\varphi(x_1)\varphi(x_2)\varphi(x_3)\varphi(x_4)$$

derive the leading order connected contributions to $G^{(4)}_\varphi$ in coordinate space, i.e. the first non vanishing connected contributions in the expansion in powers of λ. **Show your work**

b) [**0.5 point**] Draw the Feynman diagrams corresponding to each contribution in coordinate space

c) [**0.5 point**] Does the decay process $h \to H^+H^-$ occur in this theory? h is the scalar boson associated to φ and $H^\pm$ are the charged scalar bosons associated to Φ, $\Phi^\dagger$. **Show your work**

2. (3 points total) Consider the following Yukawa-like Lagrangian density

$$\mathcal{L} = \partial_\mu\Phi^\dagger\partial^\mu\Phi - M^2\Phi^\dagger\Phi + \bar{\psi}(i\partial\!\!\!/ - m)\psi + f\bar{\psi}\psi\Phi^\dagger\Phi \tag{2}$$

that describes the interaction between charged spin-1/2 fermions (ψ and $\bar{\psi}$) and charged scalars (Φ and $\Phi^\dagger$)

a) [**1 point**] Derive the equation of motion (EoM) for ψ and $\bar{\psi}$. **Show your work**

b) [**1 point**] Show that the current $J^\mu = \bar{\psi}\gamma^\mu\psi$ is conserved, i.e. $\partial_\mu J^\mu = 0$ by employing the EoMs derived in a). **Show your work**

c) [**0.5 point**] What is the global symmetry of $\mathcal{L}$ in eq. (2) associated to the conservation of J^μ according to Noether's theorem? Write the corresponding finite transformation of the fields

d) [**0.5 point**] Is there another conserved current, associated to which global symmetry?

Hint: Given two Grassmann variables, η and ξ, their derivatives satisfy:

$$\frac{d}{d\eta}(\xi\eta) = -\frac{d}{d\eta}(\eta\xi) = -\xi$$

Hint: The Euler-Lagrange equation (EoM) for a generic field ϕ reads:

$$\partial_\mu\frac{\delta\mathcal{L}}{\delta\partial_\mu\phi} = \frac{\delta\mathcal{L}}{\delta\phi}$$

Hint: The general formula for the conserved current associated to a one-generator global symmetry with parameter α reads

$$J^\mu = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \phi_a)} \frac{\delta \phi_a}{\delta \alpha}$$

where a labels all possible (transforming) fields in $\mathcal{L}$

Note: The interaction in eq (2) is nonrenormalizable in $d = 4$ dimensions

3. (3 points total) Consider the scattering process $e^+e^- \to e^+e^-$ in the Yukawa theory with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - \frac{1}{2} M^2 \varphi^2 + \bar{\psi}(i\partial\!\!\!/ - m)\psi + f\bar{\psi}\psi\varphi \tag{3}$$

a) **[1.5 point]** Draw the Feynman diagrams contributing to $e^+e^- \to e^+e^-$ at tree level Indicate which diagram corresponds to the s-channel Do all three channels s, t, u occur at tree level? If not, what is the symmetry that forbids their occurrence? **Show your work**

b) **[1.5 points]** By applying the Feynman rules, write down the amplitudes for the diagrams drawn in part a) and the total amplitude

Hints: The Feynman rule for the vertex is if